Mark Scheme 4723
June 2007

1	**(i)**	Attempt use of product rule	M1		
		Obtain $3x^2(x+1)^5 + 5x^3(x+1)^4$	A1	**2**	or equiv
		[Or: (following complete expansion and differentiation term by term)			
		Obtain $8x^7 + 35x^6 + 60x^5 + 50x^4 + 20x^3 + 3x^2$	B2		allow B1 if one term incorrect]
	(ii)	Obtain derivative of form $kx^3(3x^4+1)^n$	M1		any constants k and n
		Obtain derivative of form $kx^3(3x^4+1)^{-\frac{1}{2}}$	M1		
		Obtain correct $6x^3(3x^4+1)^{-\frac{1}{2}}$	A1	**3**	or (unsimplified) equiv
2		Identify critical value $x = 2$	B1		
		Attempt process for determining both critical values	M1		
		Obtain $\frac{1}{3}$ and 2	A1		
		Attempt process for solving inequality	M1		table, sketch …; implied by plausible answer
		Obtain $\frac{1}{3} < x < 2$	A1	**5**	
3	**(i)**	Attempt correct process for composition	M1		numerical or algebraic
		Obtain (16 and hence) 7	A1	**2**	
	(ii)	Attempt correct process for finding inverse	M1		maybe in terms of y so far
		Obtain $(x-3)^2$	A1	**2**	or equiv; in terms of x, not y
	(iii)	Sketch (more or less) correct $y = \text{f}(x)$	B1		with 3 indicated or clearly implied on y-axis, correct curvature, no maximum point
		Sketch (more or less) correct $y = \text{f}^{-1}(x)$	B1		right hand half of parabola only
		State reflection in line $y = x$	B1	**3**	or (explicit) equiv; independent of earlier marks
4	**(i)**	Obtain integral of form $k(2x+1)^{\frac{4}{3}}$	M1		or equiv using substitution; any constant k
		Obtain correct $\frac{3}{8}(2x+1)^{\frac{4}{3}}$	A1		or equiv
		Substitute limits in expression of form $(2x+1)^n$ and subtract the correct way round	M1		using adjusted limits if subn used
		Obtain 30	A1	**4**	
	(ii)	Attempt evaluation of $k(y_0 + 4y_1 + y_2)$	M1		any constant k
		Identify k as $\frac{1}{3} \times 6.5$	A1		
		Obtain 29.6	A1	**3**	or greater accuracy (29.554566…)
		[SR: (using Simpson's rule with 4 strips)			
		Obtain $\frac{1}{3} \times 3.25(1 + 4 \times \sqrt[3]{7.5} + 2 \times \sqrt[3]{14} + 4 \times \sqrt[3]{20.5} + 3)$ and hence 29.9	B1		or greater accuracy (29.897…)]

5 (i)	State $e^{-0.04t} = 0.5$	B1		or equiv
	Attempt solution of equation of form $e^{-0.04t} = k$	M1		using sound process; maybe implied
	Obtain 17	A1	**3**	or greater accuracy (17.328…)
(ii)	Differentiate to obtain form $k\,e^{-0.04t}$	*M1		constant k different from 240
	Obtain $(\pm)$ $9.6e^{-0.04t}$	A1		or (unsimplified) equiv
	Equate attempt at first derivative to (±) 2.1 and attempt solution	M1		dep *M; method maybe implied
	Obtain 38	A1	**4**	or greater accuracy (37.9956…)

6 (i)	Obtain integral of form $k_1 e^{2x} + k_2 x^2$	M1		any non-zero constants k_1, k_2
	Obtain correct $3e^{2x} + \frac{1}{2}x^2$	A1		
	Obtain $3e^{2a} + \frac{1}{2}a^2 - 3$	A1		
	Equate definite integral to 42 and attempt rearrangement	M1		using sound processes
	Confirm $a = \frac{1}{2}\ln(15 - \frac{1}{6}a^2)$	A1	**5**	AG; necessary detail required
(ii)	Obtain correct first iterate 1.348…	B1		
	Attempt correct process to find at least 2 iterates	M1		
	Obtain at least 3 correct iterates	A1		
	Obtain 1.344	A1	**4**	answer required to exactly 3 d.p.; allow recovery after error

$[1 \to 1.34844 \to 1.34382 \to 1.34389]$

7 (i)	Show correct general shape (alternating above and below x-axis)	M1		with no branch reaching x-axis
	Draw (more or less) correct sketch	A1	**2**	with at least one of 1 and –1 indicated or clearly implied
(ii)	Attempt solution of $\cos x = \frac{1}{3}$	M1		maybe implied; or equiv
	Obtain 1.23 or 0.392π	A1		or greater accuracy
	Obtain 5.05 or 1.61π	A1	**3**	or greater accuracy and no others within $0 \le x \le 2\pi$; penalise answer(s) to 2sf only once
(iii)	<u>Either</u>: Obtain equation of form $\tan\theta = k$	M1		any constant k; maybe implied
	Obtain $\tan\theta = 5$	A1		
	Obtain two values only of form $\theta,\ \theta + \pi$	M1		within $0 \le x \le 2\pi$; allow degrees at this stage
	Obtain 1.37 and 4.51 (or 0.437π and 1.44π)	A1	**4**	allow ±1 in third sig fig; or greater accuracy
	<u>Or</u>: (for methods which involve squaring,etc.)			
	Attempt to obtain eqn in one trig ratio	M1		
	Obtain correct value	A1		$\tan^2\theta = 25, \cos^2\theta = \frac{1}{26}$, …
	Attempt solution at least to find one value in first quadrant and one value in third	M1		
	Obtain 1.37 and 4.51 (or equivs as above)	A1		ignoring values in second and fourth quadrants

8 (i)	Attempt use of quotient rule	M1		allow for numerator 'wrong way round'; or equiv
	Obtain $\dfrac{(4\ln x+3)\frac{4}{x}-(4\ln x-3)\frac{4}{x}}{(4\ln x+3)^2}$	A1		or equiv
	Confirm $\dfrac{24}{x(4\ln x+3)^2}$	A1	**3**	AG; necessary detail required
(ii)	Identify $\ln x = \frac{3}{4}$	B1		or equiv
	State or imply $x = e^{\frac{3}{4}}$	B1		
	Substitute e^k completely in expression for derivative	M1		and deal with $\ln e^k$ term
	Obtain $\frac{2}{3}e^{-\frac{3}{4}}$	A1	**4**	or exact (single term) equiv
(iii)	State or imply $\int \dfrac{4\pi}{x(4\ln x+3)^2}\,dx$	B1		
	Obtain integral of form $k\dfrac{4\ln x-3}{4\ln x+3}$ or $k(4\ln x+3)^{-1}$	*M1		any constant k
	Substitute both limits and subtract right way round	M1		dep *M
	Obtain $\frac{4}{21}\pi$	A1	**4**	or exact equiv
9 (i)	Attempt use of either of $\tan(A \pm B)$ identities	M1		
	Substitute $\tan 60° = \sqrt{3}$ or $\tan^2 60° = 3$	B1		
	Obtain $\dfrac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta} \times \dfrac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta}$	A1		or equiv (perhaps with tan 60° still involved)
	Obtain $\dfrac{\tan^2\theta-3}{1-3\tan^2\theta}$	A1	**4**	AG
(ii)	Use $\sec^2\theta = 1+\tan^2\theta$	B1		
	Attempt rearrangement and simplification of equation involving $\tan^2\theta$	M1		or equiv involving $\sec\theta$
	Obtain $\tan^4\theta = \frac{1}{3}$	A1		or equiv $\sec^2\theta = 1.57735...$
	Obtain 37.2	A1		or greater accuracy
	Obtain 142.8	A1	**5**	or greater accuracy; and no others between 0 and 180
(iii)	Attempt rearrangement of $\dfrac{\tan^2\theta-3}{1-3\tan^2\theta}=k^2$ to form $\tan^2\theta = \dfrac{f(k)}{g(k)}$	M1		
	Obtain $\tan^2\theta = \dfrac{k^2+3}{1+3k^2}$	A1		
	Observe that RHS is positive for all k, giving one value in each quadrant	A1	**3**	or convincing equiv